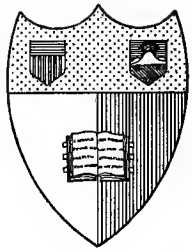

SCIENCE AND RELIGION

THE RATIONAL AND THE SUPERRATIONAL

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PREFACE

The following address aims to suggest and to sketch a new way of thinking about old things of universal interest. The major emphasis falls upon the great function of Idealization regarded in the light of what mathematicians call the method or the process of Limits. The central thesis is that this process in the domain of reason or of rational thought indicates the reality and, in part, the nature of a domain beyond, a realm superrational, and that this realm is the ultimate and permanent ground and source of the religious emotions.

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As for Knowledge, I bear her no grudge; I take joy in the pursuit of her. But the other things are great and shining.*—*Euripides.*

No doubt you will readily recall the famous dispute that occurred the other day between Protagoras and Socrates as to why it was that so understanding a people as the Athenians, though they suffered none but experts to speak in the assembly if the question before it were one of ship-building or medicine or other specific art, yet freely allowed everybody to have a say—carpenter, cobbler, tinker, sailor, passenger, rich or poor, high or low, learned or unlearned—if the matter under consideration were a question or an affair of state. Why this difference? What was the explanation? The answer, said Socrates, is this: the Athenians think that the various specific arts are capable of being taught and learned, but they are under

* Translation by Professor Gilbert Murray.

the impression that political wisdom and virtue cannot be communicated by man to man. Not so, responded Protagoras; that explanation cannot be right. For it is evident, said he, and especially so in their rational practice of punishing evil-doers, that the Athenians, like other men, think the virtues of statecraft may be acquired and taught. And in the ability to give and to receive such instruction all men have a share. For, said he, when Hermes asked whether he should distribute justice and reverence to all men or, as in the case of the technical arts, to only a few, Zeus replied: To all; I should like them all to have a share, for else cities cannot exist and the race of man will perish.

And so, according to Protagoras, the reason why the Athenians, when they met to deliberate on matters of state, were willing to hear all men, was that, in their belief, political virtue, instead of being, like an art, a privilege of a few, was an obligation of all.

It may be that a consideration analogous to that advanced by the great sophist is admissible on the present occasion. In asking you to listen to an address on science and religion by one who is not a professional

student of religion, I may perhaps plead in mitigation, and in consenting to hear me you may wish to plead, that religion is not essentially of the nature of a technical science or art, to be wholly committed to the charge of experts and specialists, but belongs rather to the general domain of human wisdom and, like political virtue, like justice and beauty, like intelligence and social order, is an affair and concern of us all.

Let me say at the outset that it is no part of my purpose to eulogize science or to magnify the importance and value of religion. It is not my intention to compare science and religion to the advantage or disadvantage of either of them. My aim is to speak candidly, quite without prejudice or partisanship, though possibly from a somewhat unfamiliar point of view, of some of the questions that arise out of the relations, or out of what many deem to be the relations, between these two great interests of mankind.

It is probable that the number of students whose devotion to science is devout enough and solemn enough to be properly described as religious far exceeds the number of those who bring to the study of religion a spirit

and method that may properly be called scientific. Yet we do not hear much about the religious study of science. We do, however, hear a great deal nowadays about what is called the scientific study of religion. What is meant or ought to be meant by "the scientific study of religion"? There exists, I believe, no little misunderstanding regarding the matter, and it may be well to begin by reminding ourselves of an important distinction. I am not going to detain you with definitions or an attempt at definition, though the distinction in question pertains to the essential natures of the great subjects we are talking about.

Anyone who has given careful study to the method and the structure of science or—what is more feasible in our brief life—to the method and structure of a representative branch of science, knows that the kind of knowledge which is currently called scientific is, in last analysis, knowledge of ideas and of the relations among them. To know a branch of science, say physics or mathematics or astronomy, at a given stage of its development, is to know a certain group of concepts together with the relations that

bind them into a logically organic whole. That is why it is said, and rightly said, that the method and the structure of science are conceptual and logical.

Now, religion, I take it, is not essentially an idea or a concept. It is not essentially a group of concepts or a group of them together with their logical interrelations. Religion is primarily, essentially and ultimately an emotion or, if you prefer, a complex of emotions. Fear, awe, reverence, love, a sense of mystery, a sense of union with a larger self, sympathy, the touch and thrill of a spiritual presence—these things and their kind are not essentially ideas, they are feelings, sentiments, emotions. In its essential nature as a complex of emotions felt in their integrity religion does not belong to the rational domain, it does not pertain to the field of logic. This is not to say that it is illogical or irrational, for these terms describe errors committed in the realm of ideas. If you wished to say that religion is hypological or subrational, I should have to object. If you wish to say that it is hyperlogical or superrational, I shall make no objection at all. What I desire to emphasize

here is that religion is not logical: it is alogical and it is as little rational as that passion of love or hate which to gain its object may fling prudence, calculation and reason all to the winds.

Is it, then, impossible to study and know religion scientifically? There is, as already intimated, a subtle and important sense, often neglected, in which it is impossible. One who has an emotion gains, in feeling it, a sense of what it is and signifies that scientific method cannot reach. This peculiar sense we may call "emotional knowledge," for the want of a better term, or knowledge-in-immediate-experience. Such knowledge of religion in its essence a scientific man may have as well as another but he cannot win it or have it in his capacity as a scientific student. If he have it he will have it by having personally the appropriate emotions. He cannot gain it by concepts and logic; he cannot formulate it in terms of them; it has no formula; it does not admit of being described or conveyed by reasoned discourse. None but a lover really knows love.

It is obvious that, on the other hand, there is an equally important sense in which reli-

gion may be studied and known scientifically. In the first place, a complex emotion admits of being, in a measure, analyzed. Such analysis and a propositional account of its results belong to the province of science. In the second place,—and this is more to the point,—an emotion, beyond the fact of its being felt, leads to manifestations that may be seen and heard: not only may it produce effects and tokens in the sensible world, exterior forms of life or living, modes of behavior, trains of events in the outward light of day, but it may invade the realm of thought, set going the machinery of logic, modify old ideas, engender new ones, transform philosophies, give birth and currency to new doctrines and views of the world. Everyone knows that the emotions which, *as* emotions felt in their integrity, essentially constitute that inner and subjective life known as religion, produce, in countless number and variety, phenomena having their locus in the outer world: physical postures, gestures and attitudes, ceremonies, customs and rites, mythologies, theologies, temples, institutions, history. Here we undoubtedly have, connected with religion as its sensible embodi-

ment, as its exterior manifestation and its counterpart in thought, a vast body of interesting and diversified material that is, in strictest sense, available for the method of science. If to this external material we add the religious emotions themselves in so far as they are susceptible of psychological analysis, we shall have before us the whole subject-matter of what may properly be called the scientific study of religion. And it is out of the study of this subject-matter by anthropologists, archeologists, philologists, psychologists, philosophers, historians and others that there has come, as you know, mainly in recent years, a copious and increasing scientific literature of religion.

Of this literature I am not about to offer a review. By many thinkers and scholars it is regarded as justifying a certain remarkable thesis respecting the relation of religion to human ignorance. Of that thesis I shall wish to speak. Before doing so, however, I desire to ask what the scientific literature of religion can tell us of religion as personal experience. What can it tell us of religion as "emotionally known" to one who has or has had immediate experience of the constit-

uent emotions? What is it competent to tell us of religion in its essential nature as certain emotions felt in their integrity? This question is answerable *a priori*, and the answer is: nothing whatever. That psychological analysis of emotions cannot tell us aught of an emotion as felt in its integrity is sufficiently evident in the fact that such analysis involves, by the very nature of its enterprise, the destruction of emotional integrity. Unlike a chemist, such analysis, though it can give us oxygen and hydrogen, is unable to give us water. As for that part of the literature which deals with the externals of religion, we need not press our question. For in their relation to scientific method, the exterior phenomena of religion are precisely on a par with the other phenomena of the external world. Most external phenomena—wind, wave and color, forms, states and transformations of matter, manifestations of light, heat and electricity—are not, at least not in our day, commonly supposed to be products of emotion. The external phenomena of religion are such products. The difference is striking. It requires some attention to discern the fact that, for scientific

method, the difference does not exist. It requires a little discernment and care to avoid confusing our "emotional knowledge" of the religious emotions with scientific knowledge of their external effects and to see quite clearly that, for science, the externals of religion are, like other external phenomena, simply objects to be observed, collocated, reduced to intelligible order, described and theorized about. It requires a little discrimination to perceive that the method of concepts and logic affords no means of feeling the origin, cause or source of its subject-matter, but that, regarding this, the best it can do is to guess and verify. We do sometimes flatter ourselves that we have "etherial" emotions, but the ether of physics is not an emotion, it is a purely conceptual thing hypothetized to account for certain facts of observation. We shall miss much if we do not see that the scientific study of the exteriorities of religion yields just that kind of knowledge of religion which the study of physics gives us of the ether. The scientific method does not require that the student of the external phenomena of patriotism or love be a patriot or a lover. For the purpose

of such a student, patriotism or love is not an emotion, it is an hypothesis. For science bent upon the investigation of certain objective facts, called facts of religion, religion is not a life, it is not a complex of felt emotions; it is, like the ether of physical theory, simply an hypothesis conceptually fabricated to bind together in intelligible order certain phenomena in the outer world.

In all this iterated emphasis upon fundamental distinctions and discriminations, I am not ignoring the intimate relationship of ideas and emotions. I admit the mingling of the two sorts of elements in our psychic life. I admit the possibility and the fact of their reciprocal genesis—idea springing from emotion, emotion from idea. I do not deny that there is even a sense in which one may speak of "conceiving" an emotion, just as I admit a sense in which an emotion born of an idea may be said to "feel" it. I do deny that an emotion and a conception of it are identical, just as I deny that an idea and a feeling awakened by it are one and the same. And so I deny that a scientific account of ideas connected directly or indirectly with the religious emotions is a doctrine of these

as felt in their integrity, just as anyone would deny that the religious emotions felt by Newton in contemplating the order and glory of the solar system are constituent parts of astronomy. I am merely insisting that in discussions about religion discrimination is quite as essential as it is in other matters. No one contends that grammar, prosody or syntax is poetry. No one contends that Burns's poem, To a Mouse, is a biological essay, that Shelley's Cloud is a meteorological disquisition, or that his Skylark is a contribution to ornithology.

Let me add that in what I have been saying of the scientific study of religion my aim has been to delimit the significance of the study. It has not been to detract in any wise from its importance and dignity. These are admittedly great.

I turn now to the thesis, alluded to a moment ago, regarding the relation of religion to human ignorance. The thesis is that human ignorance is a necessary condition and ground for the existence of religion, that religion has its lair in the unilluminated jungles of the mind, that it cannot flourish in the light of "positive knowledge," one of

the implications being that, if men and women were not ignorant, if their minds were not dark, if omniscience were a native gift or an acquisition of mankind, religion would have no source, no ground, no office and no life. The thesis is not new. Like most theses regarding matters of universal human interest, it is very old. But in these scientific times it has gained a standing and a currency that it never had before. Seemingly indicated and supported by much evidence brought to light in the scientific study of religion, the thesis is widely held to-day, not by the born sceptic, the uninformed, or the vicious, but by upright men and women of great scholarship and penetration. For example, in Professor Gilbert Murray's delightful and highly edifying work, Four Stages of Greek Religion, we meet the doctrine in its nakedness, being there told that one of the "characteristic marks," not of Greek religion in particular, but of religion in general, is that it "essentially deals with the uncharted region of human experience." Elsewhere in the volume the statement reappears in equivalent forms. If the meaning were thought to be doubtful, it is rendered

unmistakable by the following words. "A large part of human life," says Professor Murray, "has been thoroughly surveyed and explored; we understand the causes at work; and we are not bewildered by the problems. That is the domain of positive knowledge. But all round us on every side there is an uncharted region, just fragments of the fringe of it explored, and those imperfectly; it is with this that religion deals. . . . Agriculture, for instance, used to be entirely a question of religion; now it is almost entirely a question of science. In antiquity, if a field was barren, the owner of it would probably assume that the barrenness was due to 'pollution,' or offence somewhere. He would run through all his possible offences, or at any rate those of his neighbors and ancestors, and when he eventually decided the cause of the trouble, the steps he would take would all be of a kind calculated, not to affect the chemical constitution of the soil, but to satisfy his own emotions of guilt and terror, or the imaginary emotions of the imaginary being he had offended. A modern man in the same predicament would probably not think of religion at all, at any rate in the earlier

stages; he would say that it was a case for deeper plowing or for basic slag."

This same doctrine of the essential dependence of religion upon ignorance runs sinuously through the candid and magnanimous work of my friend and colleague, Professor Shotwell, on the Religious Revolution of To-Day. Witness, for example, the statement that science "renounces authority, cuts athwart custom, violates the sacred, rejects the myths." Witness the further statement that "science is moving the mystery farther and farther from the sphere of daily life and action, destroying taboos, and building up a world of rational experience; and if religion is nothing but the submission to mystery, it is doomed." Again: "The battle between science and the old religion has been a real one, and the result in any case is not the defeat of science." In Professor Shotwell's book the note is not quite so confident perhaps, and hardly so clear as in Professor Murray's deliverance; the thesis is hedged about somewhat and a little obscured by queries, conditions, ifs and buts, yet it is undoubtedly present in something more than interrogatory form and is, I think, the main

binding-thread and interest of this interesting work.

Though the doctrine is held, as I have said, by widely representative thinkers and scholars, it is not by any means a universal conviction. But it is closely allied with a conviction or a faith that is universal, and it owes to the alliance no little of its significance and much of its force and go. I mean the unquestioning faith of our time in the limitless progressibility of human knowledge. In its fulness and universality this faith is a strictly modern phenomenon, a characteristic mark of the age. *Ignoramus* we admit, but never *ignorabimus*. In the philosophic sense of the term we are all of us progressionists. We are all of us unquestioning believers in the unlimited perfectibility of man through the achievements of intellect, through invention and the discovery of truth, the advancement of science, the growth and power of knowledge. In the future and possibilities of such development, philosophers, men of science, men of affairs, the carpenter, the farmer, the grocer, all men and women, learned and unlearned, the shallow and the deep, are to-day under the sway of a faith

that was not so much as dreamed of by even the boldest thinkers of antiquity. In this regard the modern man of the street is more than a match for the greatest Athenian in the age of Pericles. What of it? I shall not here endeavor to account for this very remarkable faith, though a fairly satisfactory account of its rise would not, I believe, be difficult to give. I am not, however, at present concerned with its ground or its genesis but only with the fact itself and its implications.

It is plain enough that of these two doctrines, neither the one nor the other, when taken alone, commits one who holds it to any theory or conclusion respecting the future destiny of religion. But if the doctrines be held in combination, if we believe that religion essentially depends on human ignorance and at the same time believe in the limitless progressibility of human knowledge, then it is obvious that, as serious thinkers concerned to know the import of our convictions, we are bound to ask what is involved respecting the fortunes and fate of religion. No doubt many are prepared to answer, as indeed many have answered, by saying that religion

is doomed. Some of these, remembering the terrible things that have been done in the name of religion, rejoice in their belief in its doom; to others, valuing religion as the most precious thing in life, the prospect is sorrowful. It may be that, in pondering the matter, we shall find the rejoicing and the sorrowing premature. The question being too vast for detailed treatment here, I shall have to be content with offering you little more than a delineation.

You and I may or may not believe the doctrine that religion depends essentially upon human ignorance. But it will greatly simplify matters and facilitate the first part of the discussion if we assume, for the sake of argument, that the thesis is true. Let us, then, for the present, grant as a postulate, to use a geometric term, that religion does essentially deal with the uncharted, and, turning to the faith of our time in the unlimited expansibility of human knowledge, let us ask what ground there is in it or under it to justify hope or fear that religion is doomed. A conviction or a belief that is universal, a leading idea of an age, is always vague and is held uncritically. Therein is the secret of

its sway. When subjected to criticism, it is certain to suffer change, gaining clarity, form and definition at the expense of its certitude and power. I have no doubt that our potent and nebulous creed regarding the progressibility of knowledge is destined to illustrate this fact. And one of the evidences is that this very statement will be looked upon by many as treason in the camp of science, as a wicked assault upon the holiest faith that ever inspired the heart of an age. For, like religion, science has taboos of its own—its spirit is sacred and its hope, however extravagant, must not be touched.

Regarding the future of science many persons hold forth as if its boundless advancement were something inevitable in the nature of things, the very pet and protégé of destiny, fought for by the stars to realize on this sublunary planet, through the agency of man, a dream of omniscience, a purpose of being, older than the foundations of the world. I confess myself unable to feel such confidence and enthusiasm. The Future can not be longer than the Past has been. Here we are, not the last survivors, I hope, but certainly among the latest of a biped

race that for probably a quarter, or perhaps a half, million years has been struggling in the gloomy depths of a boundless universe of infinite complexity. And now what do we know of it? We have had indeed some precious experience, most of which has been forgotten and lost beyond recovery. Yet considering the great odds that have been against us, especially considering how short the time since our ancestors ceased to be quadrupeds and learned to carry their heads above their feet, may we not claim that, taken absolutely, the amount of our knowledge is really great and that the rate of its growth has been rapid? Of course, compared with absolute ignorance, any amount of knowledge, however small, is great. But if human knowledge and the rate of its growth are to be regarded as cosmic phenomena, then such computation of the age of our race is far from just. It is essential to bear in mind that what we are and have been—quadruped, fish or fowl, animal, plant, inorganic stuff, conscious or unconscious, ether perhaps—stretches back through countless eons of beginningless time. It is essential to bear in mind that we are thus

lineage of a past Eternity. And if we do not forget this, if we will but be at the pains to conceive, if only dimly, the innumerable succession of ages that it has taken to contrive the faculty of our little reason and to produce on this planet the flickering gleams that we call human knowledge and understanding, we shall have reason to doubt whether the production of science has really been a cosmic specialty and shall wonder rather if it may not be but an evanescent spark accidentally struck out by collision in the blundering career of an aimless and lawless world. At all events, if human knowledge be viewed as the destined aim of the course of time, no one can name a fraction small enough to express the average rate of its growth in the past.

You may wish to say, however, that all this refers to the past and need not be disputed, whilst our concern is with the present and especially with the future. Let us, then, turn at once to face the problem in its relation to coming time. Be our present knowledge regarded as little or much, it is certain that round about us on every side lies the unexplored, the region of the uncharted, the great domain which according to our postu-

late is the source and support of religion. How far does it extend? How big is it? For we are obliged to speak of it in metaphorical terms. The answer is, that in scope and in complexity of content the uncharted is infinite, and infinite of highest order. That it is so could, I believe, be demonstrated, if that were required, but I shall assume it as not being liable to denial or dispute. The meaning is that the questions to be asked and answered, the problems to be propounded and solved, the secrets to be disclosed, the truths to be discovered, the jungles to be cleared and drained, the mysteries to be dispelled, constitute an infinite multitude, uncountable and immeasurable in finite terms.

At once we must ask whether all that is contained in this transfinite domain, hid by the covering pall of our ignorance, is intrinsically susceptible of being brought into light. Is all of the unknown intrinsically knowable? We cannot be certain, but for the sake of argument I shall assume that it is. And now we must ask the question: Given any one whatever of the things in this domain of unknown but knowable things, how is it going to get known? Broadly answers

the creed of our age: The progressibility of human knowledge is limitless. I venture to say that not one in ten thousand of those who confidently repeat the creed in this or an equivalent form has been at the pains to acquire any definite conception of what the words mean. What *do* they mean? They mean, for one thing, that any closed or bounded subdivision whatever or nook of the uncharted, no matter how far it lies beyond the borders of present knowledge, will in the course of time be reached by advancing science and be explored; they mean that, whatever question be possible, no matter how remote, it will at some time be asked and answered by man. It may be so. The fact that our sense of ignorance grows with knowledge, the fact that the successful answering of one question brings a hundred new ones into view, does not disprove it. It only puts it in the light of an interesting paradox. Tristram Shandy may indeed require a year to describe the events of any given day of his life; yet, if he continues to live and, beginning with any day, continues to write the events of his life, there will be no specific event of all those in his endless

career but will at some definite time be written. But in implying that any problem whatever within the domain of the uncharted is bound sooner or later to be propounded and solved by man, our creed makes, unconsciously no doubt, a very questionable assumption. We have indeed granted that the unknown is intrinsically knowable, but the creed assumes that whatever is intrinsically knowable is humanly knowable. This assumption is extremely doubtful and cannot be granted. It is far from evident that, for the intellect of man, every specific knowable is convertible into a known. Upon a little reflection anyone should see the possibility, and what is to me a high probability, that much or even most of the knowable is only knowable superhumanly, just as much of what is humanly knowable is not felinely knowable or caninely knowable or equinely knowable, or knowable to fishes, earthworms or snails.

I shall never forget a scene I witnessed a few years ago. Walking with my wife along a street of this city in the subdued and slanting light of the setting sun, my companion suddenly blanched and veered. Instantly

the cause was evident: a large white dog viciously pursuing diagonally across the street a handsome kitten running its best for life. What to do, a moment's doubt, to help might be to hinder, too late in any case. The kitten overtaken in midstreet, about to be seized, suddenly wheels about, rears upon its hind feet, ears laid back flat, eyes flashing fire like those of a maddened hawk defending its young, strikes the dog's nose again and again, dodges, side-steps, retreats, advances, strikes again, quick as lightning leaps to the dog's side, then upon its back, runs forward and down between its ears over its face to the ground, wheels about and strikes again, repeating all the tactics, performing the program thrice in a minute, sees the enemy confused and disconcerted, turns like a flash, makes again for its home in a basement, the dog pursuing, both rush down the steps and disappear. I follow quickly, fearing the worst. Behold! The kitten has escaped by a corridor, entered a room, come to the front, and is now pressing its little white breast against the iron grating of an open window, triumphantly striking out at its angry, puzzled and defeated enemy. The

bearing of the scene is evident. Think of the prowess, the finesse of faculty, the perfect action and reaction, the wondrous instinct, intelligence, knowledge, displayed by the victor. And yet how circumscribed its range, confined within just a little sphere never to be penetrated by such an idea, for example, as that of its being mentioned in a lecture or that of an elliptic function, a flying machine or a printing press. Man, being at the top of animal intelligence in our little world, finding here no superior species with which to compare himself, assumes, quite uncritically, that whatsoever is knowable is knowable to *him*, that his present faculties in respect of kind and range require nothing but time to extend the light and dominion of *human* knowledge beyond any specific point however remote in the infinite dark of the unexplored. Nevertheless it is highly probable that, even supposing him to have endless time at his command, the sphere of his utmost attainable knowledge, though far larger than that of any lower animal, yet is as definitely limited as that of a fish or a cat. Man has some powers or faculties for knowing that the beasts do not possess. Why should

he assume that his faculties are in kind the highest possible or the highest actual? And even if they were, why assume that he has them in the highest possible degree?

At this point some acute enquirer may wish to ask whether the sphere of the humanly knowable might not be limited and yet be infinite. The answer is, It might. But this by no means implies that it would contain the totality of what is knowable. The sphere of the humanly knowable may indeed at once include an infinite multiplicity and yet exclude a multiplicity vaster still, just as, to employ a familiar illustration, the infinitude of points matching in one-to-one fashion the integers in the endless series of cardinal numbers is included in the yet vaster infinitude of all the points that constitute a line. To the possibility here recognized I hope to return at a later stage of the discussion. At present I wish to invite your attention to another alternative, namely, that the probably limited sphere of the humanly knowable may, in addition to being limited, be finite as well. I know of nothing to invalidate such a supposition. On the contrary, it is supported by considerations

of weight. If we reflect that human knowledge is of an organic unitary character by virtue of which the whole as it grows must, like a living organism, preserve a kind of symmetry involving some just proportionality of parts; if we reflect that consequently a part cannot indefinitely flourish in isolation but demands a like prosperity of adjacent parts; if we reflect that, as the parts continue to grow in number, complexity and magnitude, the danger increases of their suffering for the want of vital correlation and that the whole they constitute will as a whole be increasingly liable to the fortunes of a growing organism dependent upon cultivation but already become too vast for competent control and superintendence by a single human mind; if we reflect that paths of discovery lead always through the known, that they lengthen with the growth of knowledge, meantime multiplying their branches and intersections, becoming steeper and steeper and more bewildering; if we reflect that the difficulties of knowledge-producing investigation thus tend to increase rapidly as it succeeds and knowledge accumulates, so that already it takes the better part of a life for

a fairly good mind to gain the knowledge and master the technique essential to research in any well-worked field; if we reflect that meanwhile the capacity of the human mind to know does not increase with the demands that growing knowledge makes upon it; if we reflect that, although knowledge is an increasing function of time, the law of its growth, if indeed there be such a law, awaits discovery; if we reflect that, notwithstanding science is to-day progressing at a high and even accelerating speed, yet there are as indicated retarding forces at work; if we reflect that, on account of these, there may come a time, remote or near, when the rate of discovery shall yield to a law of negative acceleration, gradually slowing down more and more as the years go by; if we consider these things and such as these, we cannot fail to see clearly the possibility or even some probability that there exists in the nature of the case a fixed and finite limit to the possible advancement of science, a predetermined finite maximum, an outer bound that it can never pass beyond.

Is the hypothesis gloomy or depressing? Certainly not for religion, for, on beyond

that outer bound, the Uncharted, the supposed source and support of religion, would exist in all its infinitude forever. Nay, even that part of the uncharted which lies within the bound, though it would grow smaller and smaller under the continued encroachment of knowledge, could never fail quite utterly. And here, strangely enough, we see in the hypothesis good cheer for science too. For to suppose that there exists at finite remove an outer bound to the possible growth of knowledge does not imply that the growth will ever cease. It may go on forever. We know that there are countless laws in accordance with any one of which a variable may steadily grow forever, approaching asymptotically as the ages pass, but never attaining, much less surpassing, some fixed and finite magnitude prescribed in advance as the variable's superior limit. And just as a Freshman may be led to understand that a variable geometric or numerical magnitude may, conformably to some definite familiar law of growth, never cease to grow, adding increment unto increment in endless succession, and yet remain always within the compass of a finite extent, so *we* may under-

stand that the growing body of human knowledge—conveniently represented in imagination by the image of an expanding sphere—may, after a period of accelerating growth, then yield perpetual obedience to some law like that of a decreasing geometric progression, and accordingly—unless the human intellect shall fail—continue to expand forever, remaining, nevertheless, inferior to a second finite sphere concentric with the first and serving to represent the superior limit of its potential magnitude.

Would it not be possible to give the progressionist creed of our age an intelligible interpretation within this limiting sphere? It would be perfectly possible, namely, by saying that within the sphere there is no element of the unknown so remote from the center as not to be reached at some time by the spreading light of knowledge. And this interpretation is entirely true and valid, but it is very, very far from what the creed is intended to mean, for the creed is intended to cover, not merely the unknown within a finite sphere, however vast, but any point whatever of the uncharted, any definite region or zone of the unexplored.

I propose now, for the sake of argument, to abandon the hypothesis that the sphere of the humanly knowable is finite. I will suppose it to be infinite. At the same time, and again for the sake of argument, I will suppose, what I have hitherto held to be extremely doubtful, that whatever is intrinsically knowable is humanly knowable. And thus allowing our progressionist creed about knowledge the largest scope it could possibly claim, I now propose to ask, What in view of it is the prospect of religion?—not forgetting that, according to our initial postulate, religion essentially deals with the uncharted region of human experience and thus essentially depends on human ignorance. The problem is not difficult. Let us get clearly in mind the matters about which we must think. We have to think of certain things which we may call quantities or magnitudes. One of them is the infinite domain of the uncharted. Another is time. Another is human knowledge, a variable that increases with time, but whose present value or amount is finite. Finally, there is the rate of the growth of knowledge. This, too, is finite, and I assume that it will continue to be so, the abstract

possibility that the speed of advancing science may at length become infinite being too improbable for serious consideration. I fancy your thought leaps ahead of my speech, anticipating alike the reasoning and the conclusion. The infinite realm of the uncharted is to shrink as knowledge grows. Present knowledge is finite. Its rate of growth is finite and will remain so. It follows that after the lapse of any finite length of time, however long, the amount of then accumulated knowledge, though it may be immense, yet will be finite. How much of the uncharted will remain? The answer is, An amount precisely as great as before, just as if, beginning with the number one, we suppose wiped out from the endless series of cardinal numbers any succession of integers, however finitely long, the multitude remaining will be exactly as numerous as before, for the first one not wiped out may be marked 1, the second 2, and so on forever. If from a finite magnitude, a finite magnitude be taken, the original is diminished. But to diminish an infinite magnitude, it is always necessary, though not always sufficient, to take away an infinite amount. So vast is the universe

of the unknown, that knowledge may grow for any finite time at any finite rate without diminishing human ignorance one whit. That statement is indeed paradoxical, but it is nevertheless scientifically, even mathematically sound. What, then, is the upshot? It is this: the creed of the limitless progressibility of human knowledge may be allowed the largest possible interpretation and validity, generation after generation centers of knowledge may continue to multiply where new-born wonder may take its rise and put forth antennæ to feel the touch of a thrilling world, the advancement of science may proceed far beyond any prescribed point or goal, and yet the Uncharted, the source, it is said, and support of religion, will continue to surround us on every side as vast and deep and mysterious as the infinite abysses of space.

It ought to be pointed out that in winning this conclusion we have not availed ourselves of certain near-lying considerations that are graver, perhaps, than any of those adduced. We have hitherto supposed that the time which knowledge has at its command is endless. Such, however, is almost certainly not

the case. Endless time is long. In the course of the ages past, the making and unmaking of worlds has probably been as common a phenomenon as the birth and death of flies. We may as well remember that we are in a universe where, driven by incalculable forces, countless worlds of flame with innumerable hosts of attendant bodies great and small whirl and plunge, like monsters of the deep, in a shoreless ocean of space. In a universe where suns are born and die, what catastrophe may not happen in the course of time? "Time," says Virgil, "runs away with all things, including the mind." Certainly the fortunes of our own planet are bound up with those of a solar system of which everlasting stability cannot be affirmed. The famous problem of three bodies subject to Newtonian force has indeed at length been solved theoretically. Eventual collision is among the possibilities even when the moving bodies are supposed to be nothing but points. The chances of a clash are, of course, very much greater when the moving bodies, instead of being mere points, are in reality as huge as Earth or Mars or Jupiter. The far more complicate problem of n bodies, where n is

greater than three, has not been solved even theoretically. There is every reason to suppose, however, that the danger of disaster increases with the increase of *n*. Of several hundred solar planets our own belongs to the group of the major eight. Now, we should not forget that human knowledge is a plant of Earth, and in talking about the possibility of its limitless growth, it is but fair to remember that the race of man, with the huge rushing ship that bears him along shifting courses amid swift-moving planets and stars, may be destined to perish, sooner or later, in a crushing collision of worlds. It is true that in such a catastrophe religion, too, would perish, but the uncharted would survive.

Neither should we fail to reflect that the case would be essentially the same if, instead of collisional destruction of our planet, the sun were to die and the life of mother Earth were to perish in snow and ice. Recent physical discoveries in relation to radium and to the constitution and energy of atoms have indeed much mitigated the confidence with which not long ago eminent men of science, Lord Kelvin for example, were wont to pre-

dict as a far-off event so luring a doom. Yet we have to allow that an icy winding up of sublunary affairs is more than a mere possibility. You may perhaps recall Anatole France's graphic description of what would happen in that event. "When the sun goes out—a catastrophe that is bound to be—mankind will have long ago disappeared. The last inhabitants of earth will be as destitute and ignorant, as feeble and-dull-witted, as the first. They will have forgotten all the arts and all the sciences. They will huddle wretchedly in caves alongside the glaciers that will then roll their transparent masses over the half-obliterated ruins of the cities where men now think and love, suffer and hope. All the elms and lindens will have been killed by the cold; and the firs will be left sole masters of the frozen earth. The last desperate survivors of humankind—desperate without so much as realizing why or wherefore—will know nothing of us, nothing of our genius, nothing of our love; yet will they be our latest-born children and blood of our blood. A feeble flicker of the regal intelligence of nobler days, still lingering in their dulled brains, will for a while yet enable them

to hold their empire over the bears that have multiplied about their subterranean lurking-places. Peoples and races will have disappeared beneath the snow and ice, with the towns, the highways, the gardens of the old world. With pain and difficulty a few isolated families will keep alive. Women, children, old men, crowded pell-mell in their noisome caves, will peep through fissures in the rock and watch the somber sun mount the sky above their heads; dull yellow gleams will flit across his disk, like flames playing about a dying brand, while a dazzling snow of stars will shine on all the day long in the black heavens, through the icy air. This is what they will see; but in their heavy witlessness they will not so much as know that they see anything. One day the last survivor, callous alike to hate and love, will exhale to the unfriendly sky the last human breath. And the globe will go rolling on, bearing with it through the silent fields of space the ashes of humanity, the poems of Homer and the august remnants of the Greek marbles, frozen to its icy surfaces. No thought will ever again rise toward the infinite from the bosom of this dead world." For inviting you to glance at

so somber a picture I offer you no apology but this: namely, the questions and creeds we are discussing oblige us as candid students to try to look afar.

It remains to mention briefly another possibility, a classic one that has haunted the minds of thinkers from the earliest times, figuring in speculation from Empedocles and Epicurus down to Herbert Spencer: a possibility that looks both forward and backward, embracing at once the total succession of events that Time can present. I mean the possibility that our universe is what mathematicians know as a cyclic group. I may intimate the character of the great conception clearly enough perhaps by means of a simple, tiny, trivial example. Imagine three things, *a*, *b*, *c*, to be so operated upon that, no matter in what order they be arranged, each of them shall continue to be replaced by the one that follows it. Thus we shall get *a b c*, *b c a*, *c a b*, *a b c*, and so on in endlessly repeated cycle, like that of morning, noon, evening, midnight and morning again, like that of the seasons, like that of seed, plant, fruit and seed again, like countless other imperfect illustrations to be

found, more or less disguised, everywhere in the world about us. Now the speculation of many thinkers has been that the cosmic flux, the stream of the world's events, instead of moving endlessly *forward*, forever presenting the *new*, may be in fact a cyclic stream, completing a circuit in a long but finite period of time and so presenting in unchanged order again and again, without ceasing, all and only things and events that are extremely *old*, having already traversed the self-same round infinitely many times. Whether the speculation be true or not, this great concept of the Cosmic Year with its doctrine of "nothing new" has long since won for itself, like the Platonic doctrine of reminiscence, the glory of living expression in the enduring form of verse, as in the fourth eclogue of Virgil, for example, in the mighty poem of Lucretius, and in Chidher, the beautiful poem of Rückert. Time does not permit us to dwell upon the manifold implications of this hoar and luring hypothesis, but in its bearing upon our subject one thing at least is evident: even if it were supposed that in such a cyclic cosmic scheme knowledge might, in the course of a given cycle, explore

the uncharted completely and that thereupon religion might cease to be, yet, that cycle once completed, knowledge itself would have vanished and, again starting from non-existence, it would be obliged, along with new-born religion, to repeat again the same old tale of strife and struggle up the steep and winding course of cyclic evolution. "Every art," said Aristotle, "and every kind of philosophy have probably been found out many times up to the limit of what is possible and been again destroyed." In this connection I must allow myself one additional word. A very large part of the uncharted consists of what we do not know of the Past, and unto that part are added, with each passing hour, increments compared with which the recoveries of modern historical research are infinitely trifling. Even under the most favorable hypothesis, namely that of the cycle and cosmic year, there is no evidence or prospect whatever that the great and growing infinite body of what lies buried in the dark of the ages gone will ever come forth into the light of human knowledge.

Herewith I close the first part of my argument. We have contemplated a wide

variety of considerations, some of them essentially minute and subtle, others open and vast like the infinite secularities with which they deal. All of them seem to converge upon an inevitable conclusion. It is that the great creed of our age regarding the limitless progressibility of human knowledge admits of no interpretation to justify hope or fear that religion is under a "steadfast ordinance of doom," even when we grant, as for the sake of argument we have hitherto granted, that religion "essentially deals with the uncharted" and thus essentially depends for sustenance upon human ignorance.

But that postulate is to be no longer granted. It is to be now withdrawn and henceforth denied as being contrary alike to reason and to fact.

It is obvious that for an omniscient being, for one knowing all, there could be no such thing as a region uncharted, no such thing as an "un-understood." I am not about to affirm or deny the existence, possibility or actuality of such a being. I know a little something of the difficulties involved and I do not intend to hazard the issue of this discussion by dogmatic and categorical

statements that many of you might challenge and that in any case are inessential to the support of my argument. In relation to this matter I shall confine myself to the hypothetical, keeping well within safe territory by saying only, in mathematical fashion, that, "if so and so, then so and so." And now I say that, if religion depends essentially upon ignorance, an omniscient being could not be religious. Yet religion, being a fact in the world, is one of the things that an omniscient being, in order to be omniscient, would have to know and know fully and precisely, in generality and detail, in its hidden recesses and its open reaches, in its every light and shade and color and tone. In knowing all about and of religion, an omniscient being would know, among much else, just what religion signifies and is to one who feels the religious emotions in their unanalyzed integrity. In other words, such a being would have what I have called "emotional knowledge" of religion and, being omniscient, would have it in every respect precisely *as* it is for any specific man or woman. But to feel a religious emotion in its integrity is to be religious. Accordingly an omniscient being,

in feeling the religious emotions as they must needs be felt to know them *as* they are known to a man or a woman in feeling them, would thus be an essentially religious being, one having genuinely religious experience. And thus it is seen that the doctrine of the essential dependence of religion upon ignorance plainly involves a logical contradiction, and this means that it is, as said, contrary to reason.

I now maintain that the doctrine is, as also said, contrary to fact. In order to clear the ground and avoid, if may be, the possibility of misunderstanding, let me begin by making certain concessions and avowals. It goes without saying that the forms of religion, its external embodiments in rite, institution and doctrine, vary very greatly with time, place and circumstance. I do not dispute that, among the circumstances that have fashioned these forms and that continue to mold and modify them, by far the most effective determinant is the growth and dissemination of knowledge. I do not deny that, owing chiefly to the influence of advancing knowledge, many forms of religion have passed away in the long course of

time and that many of its forms are to-day in process of passing. If we are to speak of religions instead of Religion, I admit that the light of knowledge has destroyed religions and is doing so to-day. I do not deny that, if religion be identified with its forms, if we refuse to distinguish between its life and the visible manifestations of its life, then we are obliged to say: As knowledge advances, religion must recede; the twain are incompatible. But I deny the justice of the supposed identification. The form of a life may undergo striking transformation whilst the life still remains substantially invariant, without breach of continuity, without declination of vigor or any tendency to degradation or decay. On the contrary, change of form may signify development, waxing vitality, continuing adjustability to changing environment, increase of level, amelioration, augmentation of prosperity and power. Consider the sensible or outward manifestations of Wonder, for example, or Curiosity. They continually change, and the forms of enquiry, culture and education vary greatly, with time, place and circumstance, especially responding to the altering demands of in-

creasing knowledge, but no one contends on such account that the animating life of the changing forms is pursuing a destined course to extinction. Nay, Knowledge itself, by virtue of a principle inherent in it, is constantly undergoing transformation of its external form and body, but we do not argue that, therefore, it is the nature of knowledge to be drying up its own springs and approaching death in a desert.

But, one may ask, admitting all this, is it not true that the effect of knowledge upon religion goes much deeper than its forms, its exterior manifestations, its sensible embodiments? Does not knowledge, does not the light of ideas, penetrate to the very core of religion, affecting the very emotions themselves, the feeling of which in their native integrity is religion's life, religion itself? The answer is, Yes. I do not deny it. I admit that the light of knowledge, the radiance of ideas, reaches the religious emotions as it reaches other emotions, affecting them profoundly, controlling them in a measure, helping to determine the occasions of their rise and subsidence, giving them new directions, changing their temperatures,

velocities and moments, altering their intensities, emphases and colors. I admit the deep and complicate interplay and reciprocal action of ideas and feelings, of emotions and knowledge; I admit that the two natures are alike subject to development and evolution. But I deny that the affectional nature, though it is modified by intellect, is destroyed by it: I deny that knowledge destroys emotions. Who will submit a list of the emotions that have been destroyed by a hundred or a thousand years of advancing knowledge? Charles Darwin, it is true, has told us that through long exclusive addiction to the study of science, he lost his joy in poetry, but is not that loss to be ascribed to the atrophy of a faculty long unused and the hypertrophy of other faculties rather than to any essential incompatibility between science and song? Nearly all great men of science, Darwin included, have been, potentially and essentially, poets. For poetry, the most permanent and ubiquitous influence in life, pervades science too; there is a poetry of sheer ideas, and in the light of pure thought there gleam ideal architectures to galvanize the spirit to the highest mood. What knowledge destroys

is ignorance but not emotion. If, largely through the effect of increasing knowledge, I learn to prefer the violin to the tom-tom, or a symphony of Beethoven to the rude melody of a savage, I do not infer that knowledge is destroying in me the emotional life of music. If, largely through gain of knowledge, I come to prefer the Mona Lisa to a rude portrait of an Indian squaw or learn to value the Parthenon more highly than some rude African temple, I do not conclude that knowledge is incompatible with the emotions of beauty and worth. If, through acquisition of new ideas, the Esquimau shall cease to feel it a pious duty to kill his father and mother ere decrepitude disqualifies them for a happy life beyond, and learns to feel instead that he ought rather to lengthen their years even at the expense of his own, we shall hardly infer that his ethical emotions are in process of extinction. In all such cases what we infer is, not decadence, but amelioration. I contend that so we should think of the religious emotions. As science advances, as knowledge penetrates and spreads, these change but they do not die. Their objects change. Sympathy with

a clique or a club or a tribe or a local church may grow, under the influence of growing knowledge, into a living sense of universal brotherhood including our kin, the beasts. The things we fear, the things we love, the things that awaken our reverence and awe, the things that mystify or thrill, these may change and pass, but out of the infinite resources of life others replace them, and the emotions themselves, grown wiser and purer in accordance with a law of spirit, survive all the vicissitudes and continue to live and flourish in ampler relation and higher form.

To say that ignorance is the *fons et origo* of the religious emotions, to say that religion has its lair in the unilluminated jungles of the mind, is simply not true. A far deeper philosophy is required. The cosmic times and spaces of modern science are more impressive and more mysterious than a Mosaic cosmogony or Plato's crystal spheres. Day is just as mysterious as night, and the mystery of knowledge and understanding is more wonderful and awesome than the darkness of the unknown. No one that has seriously sought to understand knowledge or to know the ultimate nature of understanding;

no one that has tried to penetrate the secret recesses of logical implication, to thread the inmost mazes of ideal relationships and to feel in their essence the subtile affinities of thought; no one that has keenly realized the indissoluble interlocking of thought with thought independently of temporal circumstance or human purpose or will; no one that has clearly beheld in the silent light of meditation great cathedrals of doctrine poised in eternal calm above and upon the spiritual basis of a few select ideas; no one that thus has had a vision or even a glimpse of the abiding reality under the changeful garment of the world: no such person can fail, I think, to perceive and to feel that the supreme religious emotions of reverence and love and awe, so far from depending upon ignorance, are but elevated, amplified and deepened by the mysteries and the wonders more and more disclosed in the brightening light of knowledge. Not in the uncharted but in the charted, not in the unknown but in the known, not in ignorance but in knowledge, it is there, in the light, that we shall find, if we look, an ever-deepening well of wonder

and thrill and mystery and reverence and awe.

Finally, we may admit the fact itself and seek its explanation. If we do so, if we enquire why it is that the light of knowledge, instead of being inimical to religion, is destined to be its purer and fuller source, my answer, which is the culminating thesis of this address, will have, I venture to believe, a significance beyond its bearing on religion. I desire to submit it for your candid consideration. It is commonly supposed that the sphere of our experience and psychic life is composed of two zones, the domain of Sense and above it the domain of Reason. If we disregard the lower zone, if we take away that great subrational domain which we share jointly with the beasts, it is commonly supposed that what remains, though but little of it has been actually explored, yet is under the potential dominion of reason, intrinsically open, that is, to thoroughgoing conquest and occupation through the rational means and processes of concept and logic. That creed I am convinced is false. I maintain, I believe upon scientific ground, that the domain of reason, the great realm

of whatever is open to exploration by rational means, is infinitely far from containing all that lies above the basal zone of sense. My thesis is that the Rational implies and reveals the Superrational, and that the latter is the source of influences which, if but dimly seen, yet are keenly felt in the deeper centers and higher moods of life. I contend that, as rational knowledge advances, as the light of reason spreads and intensifies, it more and more reveals evidences and intimations that over and above reason's domain, overarching and compassing it about, there lie regions of reality unto which the rational nature of man indeed aspires, approximates and points, as unto its ideal and over-world, but which it can never attain, much less subdue to the ways of common knowledge, or the familiar forms of thought. Even the darkest mind must needs have, it seems, at least some dim sense of such a region or realm whence proceed vibrations, so to say, that find way across the far-off borders into reason's realm and, breaking against the forms of being there, kindle into strange radiance of a higher world.

Here is no question of the uncharted in

ordinary sense; it is not a question of a realm where logic has not yet been; it is a question of a realm into which logic cannot go, of a realm lying beyond the bounds eternally fixed by the principles of logic itself. What and where are the evidences for the existence and reality of such an outlying zone of being? And what is their form and guise? The evidences, I have said, are to be found in the rational domain itself: the intimations, the indications of superrational reality shine more and more, I maintain, in the brightening sheen of reason itself. They present themselves, as we shall see, in the manner and guise of idealizations, in the ways of winged pursuit of the "ever-flying perfect"; they are figured forth in the form of unending sequences or series amenable to logic, traversing the rational domain, and indicating, by the laws of their march through the world of reason, limits that lie beyond. Can the evidences be clearly produced here in court? Not, I fear, without some slight recourse to the use of technical terms bearing unfortunately the unloved savor of mathesis. Innocence of mathematical technique is doubtless venial in all but the

professed mathematician. To surrender, however, or run away before every token of precise and rigorous thinking is the shame of culture. I say it the more boldly here, being of course well aware that this occasion cannot illustrate that reproach. Certainly in speaking to an audience of Phi Beta Kappa one need not fear or hesitate to serve oneself, as occasion demands, with an idea drawn from the science of Freshman days, especially when its significance, though preëminent in mathematics, pervades moreover, did we but see it, the whole of our mental life, from the prosaic activity of the counting-house to the airiest spirit of song.

How beautiful a thing is a circle. In a circle let there be inscribed an equilateral triangle, then a regular hexagon, then a polygon of a dozen sides, and so on forever, going from step to step of the summitless scale by the simple device of ever doubling the number of sides. Infinitely many are the polygons so obtained. Each of them has a certain size, a certain area; the first is the smallest, the second is next, and so on *forever*. Let us suppose all these areas arranged in a series, in the order of size, beginning with the

smallest. Indeed, they are already so arranged. There now lies before us for our contemplation a literally endless sequence of ever-increasing terms, of ever-increasing polygonal areas. In respect of size, these approach nearer and nearer, as close as we please, to the size of the circle's area, yet they remain inferior to it forever. And so we say, in technical language, that the circle's area is the sequence's *limit*. It is important to note that the sequence's limit is not a term in the sequence, for all these terms are polygonal areas—shapes bounded by polygons—but that of the circle is not, for the circle is not a polygon. The totality of all areas whatever that are bounded and shaped by polygons I shall call the Domain of Polygonal Areas. Within that domain are contained, among many other polygonal areas, all the terms of our sequence but not the sequence's limit: the circle's area does not belong to the domain of polygonal areas but is a thing upon its border. The terms of the sequence may be viewed as the steps of a path beginning with the first term and thence proceeding on and on, within the domain of polygonal areas, step after step endlessly,

on and on out towards the border, getting closer and closer to it, just as near as we please, and, though never attaining it, yet indicating by the law of approach an unmistakable something that lies thereupon, namely the circle's area. Here, then, we have a clear presentation, within a given domain, of something that is not within: we have a clear presentation, by the law of an inner sequence, of a limit on the rim—of an ideal, if you please, which, so long as we operate within the domain, may be aspired unto, approached and pursued forever, but can never be attained. In this simple and familiar example we have a miniature pattern of what is to be the scheme of our larger thought. Similar examples abound and others of them would serve just as well. It will, however, be sufficient, I think, to cite but one more, for "the clue, familiar to our hand, lengthens as we go, and never breaks."

I will take my second example in the field of Number. Consider the whole numbers together with our everyday, familiar, vulgar fractions. These whole numbers and fractions together constitute a domain which for the purpose of this discussion we may call the

Domain of Common Number. Let us now agree to operate within this domain and see if we can find there, within, any certain presentations or indications of definite things that are not within. Here, as in the foregoing illustration, our instrument and guide will be what we have called a sequence. In sequences our chosen domain of operation is immensely rich. For suppose the numbers arranged in the order of size beginning with zero. Then, for example, the ensemble of all those numbers that precede any given number of the domain will constitute a sequence. I now invite you to think of a very obvious and special sequence, one however that represents an exceedingly important type, for there are many types. The sequence I wish you to consider is composed of all the numbers whose squares are each of them less than the number 2. Observe that, just as the squares of the sequence-numbers approach as near as we please to 2 but never reach it, so the numbers themselves, the roots of the squares, approach as near as we please to the square root of 2 but never reach the root. Accordingly we say, again using technical speech, that the square root of 2

is the sequence's *limit*. As in the preceding illustration, so also here it is important to observe that the limit is not a number in the sequence, being indeed neither a whole number nor a vulgar fraction: the limit is not a thing within the domain of operation, the domain of common number. The sequence itself lies wholly within, but its limit is on the border. With respect to the domain, the limit is a sheer ideal, a creature of idealization, an ever-flying perfect which no pursuit, however tireless and swift, along the path or course of the sequence leading towards it, can ever overtake.

All this, you may wish to say, is sufficiently subtle and is doubtless scientific, but what, pray, can be its bearing upon religion? I think we may see that its bearing, not only upon religion, but upon a right understanding of our psychic life in general, is wide, weighty and deep. In the two familiar illustrations that we have drawn from the great fundamental doctrines of Number and Space, we behold, in its simplicity, purity and perfection, a situation that, far from being confined as commonly supposed to Mathematics, is really present, in more or less

disguised and imperfect form, everywhere throughout the range and scope of our mental activity and life. "What must be said may as well be said twice o'er." Adequate statement economizes argumentation. And so, in the interest of elaboration and emphasis, for the matter is very important, I repeat that the situation in question is literally omnipresent. I say that, if we will but look attentively, we shall find that domains, similar in general structure to those I have pointed out, exist here, there and yonder in countless number and variety, constituting the vast and complicate world of our mental life; we shall find that each of the domains is carved out and its boundaries determined by the nature of its content, by the kind of objects or spiritual entities that make it up, so that two domains differ and are indeed two by virtue of a difference in the characteristic properties of their respective contents; we shall find that two domains are thus the actual or potential homes of two differing doctrines and that generalization consists in noting what is common to them; we shall find that the entities of a given domain, the objects of sense or of thought in which its

content consists, are, in general, disposed or disposable in the order of endless sequences; we shall find that, in most cases, such sequences serve as paths or tracks on which the mind, operating in the domain, may pass from a given object in it outward towards the border; we shall find that, though the border be not thus attainable, though our approximation to it be at best but asymptotic, yet the law of approach indicates the existence and in part the nature of objects upon the border; we shall find not only that the border of a domain is thus indicated from within by the so-called "method of limits," by the process of idealization, but that the border of a domain is itself a domain, a domain whose content differs from that of the former in some essential respect, as the border domain of curves, for example, differs from that of broken lines; we shall find that the domain thus serving as the border of a given domain, far from having to be as an "imaginary line" bounding a field, is often a vaster and more complex affair, greater in wealth of spiritual content than the realm it serves to bound, just as, for example, the domain of polygonal areas is inferior to its

border of curved areas, and just as, for another example, the domain of common number is far less extensive and rich than the realm of numbers that constitute its rim. Such, in brief, are a few indications of what appears to be a just and helpful vision of the make-up and ways of our psychic life:

Realms of spirit everywhere,
Nest in nest, lair in lair:
Ideals within are reals without,
Encompassing fields are compassed about.

Nothing short of a pretty large volume would suffice to present in its full significance the indicated view of the world. My present concern is to relate the view to the question of the existence and reality of a domain of superrational being. Consider first the subrational domain—the great fundamental zone of Sense—in its relation to the field of the rational—the great domain of Reason. I am not going to discuss the general question of reason's dependence upon the facts of the sensible world. What I wish to point out here is this: namely, that the countless phenomena in the world of sense form and present to us there innumerable series or

sequences having for their limits ideal things that belong only to the world of reason: the realm of things *per*ceived has for its border the realm of things *con*ceived, the world of things perceived being like an immense and diversified map glistening everywhere with endless courses or tracks approaching asymptotically an ideal region beyond. The evidence abounds on every hand, constituting a genuine embarrassment of riches. For example, in the world of sense, matter presents itself in various degrees of permanence of form, gaseous, liquid and solid; but, as Tresca and others have shown, what we call solids can be made to flow in jets from the bottom of vessels, like a liquid: perfect solids, like perfect gases, are nothing but limits, sheer ideals without existence in the world of sense, pure concepts in the domain of reason. For another example, consider the beautiful phenomena of crystallization. In respect of formal perfection the multitudes of crystals found in the sensible world constitute a variety of sequences approximating more and more nearly to certain forms, as the cubic, the tetragonal, the orthorhombic, and others, but these forms are never actually

realized in the subrational world of sense: they are there but indicated as limits beyond, as ideals having existence only in the domain of concepts, the world of reason. A perfectly symmetric tree exists only in the rational world, it is but a dream, an ideal thing indicated as the limit of a sequence found in the forests of the world of sense. So, too, with harmonies of sound: harmonies that are heard are imperfect, but the ideal pursued by them, the flying goal of their aspiration, the dream of their dreams, is not a sound; it belongs to the world of reason; perfect harmony is a thought, silent as the music of the spheres. If to the cells of animals and plants, the rounded pebbles of the beach, drops of mercury and corpuscles of blood, we add Plateau's beautiful globules of oil, we shall be able, with these exquisite objects of the sensible world, to constitute an endless sequence having for limit the form of a perfect sphere, but the sphere is not a thing in the realm of sense, it is a concept, a thought dwelling apart upon the overarching border. It is needless to cite further examples; their number is as the sands of the sea. They bear in combination overwhelming

witness to the fact that the field of what is rational, the realm of concept and logic, the domain of Reason, is fundamentally, in the sense of the terms already made clear, the great limit, ideal or border of the subrational domain and basal zone of Sense.

What of it? I shall answer at once, for I do not intend to pause here in order to show how the same scheme of things obtains also *within* the rational domain itself, finding infinite illustration there in and among its countless subdivisions or subdomains. Proceeding at once to my thesis, I maintain that, just as with respect to the subrational domain of sense, the rational domain is a limit, ideal and overworld, so we may find in the rational realm itself clear and ubiquitous evidence of the existence aloft of a realm superrational, the limit, ideal and overworld to the world of reason. The thesis obliges us to produce in the world of logical thought rational sequences that, by the law of their formation and progress, approach and betray as a border-domain a region of reality from which the dominion of logic is forever barred. The obligation is not difficult to discharge. The notion, for

example, of what is called a class of things lies at the very foundation of logic, being literally omnipresent in the realm of reason. Nothing can be easier, nothing indeed is more common or familiar, than to form in thought a sequence or series composed of more and more comprehensive classes, and having for limit the entire universe of things. Now, it is easy to show that this limit, this vast ideal, the universe regarded as a class of all things, does not belong to the content of the rational domain. For one thing, such a class would have to include itself as a member of itself, a phenomenon that cannot occur under the reign of familiar logic. Indeed, if we attempt to apply syllogistic process to the universe as an all-inclusive class, we are immediately led into the flattest contradiction. Let it be tried. Let us say that the universe U is the class of all classes. Then to say that a class C is a member of U is to say that C is a U (as we say, for example, that Socrates is a man). Now to say that a class x is a U implies that x is not an x, since a class is not a member of itself. Hence to say that U is a U—that the universe is a universe—

implies that U is not a U—that the universe is not a universe: an absurdity exquisite enough for the most fastidious and plain enough for the most obtuse. Here, then, we have a sequence composed of rational terms (classes that are amenable to the processes of logic), a sequence traversing the domain of reason and indicating a limit on the border, an ideal belonging to an overworld. It is futile to deny the limit's existence. That there is a universe is a fact we cannot escape. The statement of Lucien Poincaré that "we know nothing of the universe as a whole" is quite true if by "know" we mean know rationally. Yet the universe exists as a subject in the discourse of even a Poincaré.

Essentially the same result will follow if, instead of the notion of Class, we deal with the other great fundamentals in logic and reason. I refer, of course, to Propositions and Relations. Every one knows that the joint affirmation of two or more propositions is an additional proposition. Let us now think of a sequence of joint affirmations becoming ever more and more inclusive in such a way that the indicated limit of the sequence will be the joint affirmation of all

propositions whatsoever. It is obvious that the limit indicated by such a sequence of rational terms does not fall within the rational domain but is something beyond, for, within the domain, the joint assertion of two or more propositions is a new proposition, whilst the limit, in affirming simultaneously all propositions, must at the same time affirm itself. The situation, in technical speech, is this: within the range of the rational, the logical sum of two or more propositions is a new proposition, constituted by the sum but not contained in it, whilst the sequence of such sums has for limit, out upon the rim of the range, a superlogical sum, a sum that, in embracing at once all propositions, must embrace itself. I need not tarry to show that a like superrational phenomenon is in similar manner betrayed in the rational theory of relations.

That the Rational implies and reveals the Superrational, that rational processes involve and intimate a region of reality beyond their range, is thus evident in the most central matters of logical thought, in those primal concerns where the light of reason is clear and cold and steady and dry. We are

far from being compelled, however, to depend solely upon evidence so austere and chill. The domain of reason is thronged with phenomena bearing confirmatory witness of a warmer kind. We familiarly speak of rational knowledge, for example, as having a kind of extent, and, by way of measure, we commonly figure it to ourselves as enclosed in a circle or a sphere. There is no better way, for all our thinking is, in last analysis, in expression at all events, metaphorical, symbolic, diagrammatic. We think of such spheres as theoretically ever increasing in size, in volume, forming thus an endless sequence of ever larger and larger spheres of potential knowledge, the assumed law of expansion being such that the sequence's limit, the implication of the law, the ideal indicated by it and forever pursued, is Omniscience. Omniscience, however, is obviously not one of the spheres of the sequence. These spheres are, each of them, immersed in ignorance, enveloped by the unknown, each of them is as a globe of light surrounded by darkness, each of them is an arena for the proper activity of concept and logic, the radius extends and the surface expands under stress of

rational processes occurring within. But omniscience is not something immersed in the unknown, it is not a globe of light shining in the dark, it is not an arena for concept and logic. "Does God think?" asked the Persian pupil. The master replied: "Man thinks because he does not know, God knows and so he does not think." The point is obvious: thinking implies the unknown, omniscience does not; thinking is a perpetual campaign for light, the way of its radiant march is an endless course traversing the domain of Reason; omniscience, the limit, lies beyond; knowledge is rational; omniscience is superrational: it is knowledge supernalized, the ineffable glory of an Overworld.

For final witness, a mighty witness, to the truth of my thesis let me remind you of the superrational limit shining above the summitless scale of excellence so wonderfully portrayed by Plato in his immortal vision of Beauty and Love.

"These are the lesser mysteries of love, into which even you, Socrates, may enter; to the greater and more hidden ones which are the crown of these, and to which, if you pursue them in a right spirit, they will lead,

I know not whether you will be able to attain. But I will do my utmost to inform you, and do you follow if you can. For he who would proceed aright in this matter should begin in youth to visit beautiful forms; and first, if he be guided by his instructor aright, to love one such form only—and of that he should create fair thoughts; and soon he will himself perceive that the beauty of one form is akin to the beauty of another; and then if beauty of form in general is his pursuit, how foolish would he be not to recognize that the beauty in every form is one and the same! And when he perceives this he will abate his violent love of the one, which he will despise and deem a small thing, and will become a lover of all beautiful forms; in the next stage he will consider that the beauty of the mind is more honorable than the beauty of the outward form. So that if a virtuous soul have but a little comeliness, he will be content to love and tend him, and will search out and bring to the birth thoughts which may improve the young, until he is compelled to contemplate and see the beauty of institutions and laws, and to understand that the beauty of them all is of one family, and that

personal beauty is a trifle; and after laws and institutions he will go on to the sciences, that he may see their beauty, being not like a servant in love with one youth or man or institution, himself a slave mean and narrow-minded, but drawing toward and contemplating the vast sea of beauty, he will create many fair and noble thoughts and notions in boundless love and wisdom; until on that shore he grows and waxes strong, and at last the vision is revealed to him of a single science of beauty everywhere. To this I will proceed; please to give me your very best attention.

"He who has been instructed thus far in the things of love, and who has learned to see the beautiful in due order and succession, when he comes toward the end* will suddenly perceive a nature of wondrous beauty (and this, Socrates, is the final cause of all our former toils)—a nature which in the first place is everlasting, not growing and decay-

* Of course there is no "end"; if Plato had known and employed the language of modern mathematical analysis, he would have said, not "when he comes toward the end," but, as he more and more *approaches* the *limit;* for this latter is plainly his thought.

ing, or waxing and waning; secondly, not fair in one point of view and foul in another, or at one time or in one relation or in one place fair, at another time or in another relation or at another place foul, as if fair to some and foul to others, or in the likeness of a face or hands or any other part of the bodily frame, or in any form of speech or knowledge, or existing in any other being, as for example, in an animal, or in heaven, or in earth, or in any other place; but beauty absolute, separate, simple and everlasting, which, without diminution and without increase, or any change, is imparted to the ever-growing and perishing beauties of all other things."

Such in brief is the holy vision of the prophetess of Mantineia: a vision of achievement unending—wisdom and love mounting a summitless scale of excellence, level above level forever, through the world of Sense and the world of Reason, towards a perfection and a glory Supernal. For it is evident that beauty absolute, separate, simple, invariant and everlasting, transcends alike the stream of sense and the established ways of thought.

Finally, it requires but little reflection to see that, *mutatis mutandis*, we are confronted by essentially the same situation in all the cardinal sub-domains of the rational understanding: whether it be time that we contemplate, or power, or ubiety, or order and law, or degrees of indetermination, or right, or concord, or virtue, or joy—in every category where the laws of reason reign we find that the great process of Idealization points aloft to some form above the laws: we find that—like the Class of all Classes, like the Joint Affirmation of all Propositions, like the Logical Sum of all Relations, like Omniscience, like Beauty absolute—so, too, Eternality, Omnipotence, Omnipresence, Necessity or Fate, Unconditioned Freedom or Self-determination, Perfect Justice, Universal Harmony, the Goodness of God, Felicity Divine, and many other supreme ideals and supreme perfections of rational experience and thought, are all of them forms of Being absolute, constituting an Overworld, a realm Superrational. That realm supernal, flying canopy of Thought, far off fathomless sky of the Rational spirit, thus revealed as the supreme implicate of Reason and Sense, will

henceforth ever as in the past shed in human lives, whether they be schooled or unschooled, a mystic radiance like the "obscure clarity that falls from the stars." May I in closing summarize my thought in verse?

Beneath the whole a basal zone:
Sense supports not Thought alone,
For ways of Reason point above
Towards Perfect beauty, wisdom, love:

High and vast beyond compute,
A realm of Being absolute,
Supernal source of lights that glow
In radiant tremors felt below.

Reason's glory is in her Dream,
Her highest Truth and Worth supreme
Intimate and half reveal
What they are, in what we feel.

Not in the jungles of the mind
Religion's well-spring shall we find.
Not of Darkness is her might
But of the mystery of Light.

Nay, Thrill and Awe with Grace and Love
Eternal flow from Founts above
The vale of Sense and Thought's confine
To make our common life divine.

Illusion all? How are we blind
To deem illusion of the mind
The holy Light by which we see,
The sheen of Ideality:

The Light and Soul of what we mean,
What is Felt in what is seen,
The hid Intent of thought, unfurled,
The Glory of the Overworld.

To debate the "existence" of such a world were a vain dispute. In some sense, whatsoever quickens, lures and sustains, exists. Aspiration is not mocked. Reason's unattainable ideals are the light-giving Æther of Life. Therein is the precious and abiding reality of the Overworld.